

SUPERPOTENTIAL FROM BLACK HOLES

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ABSTRACT

BPS monopoles in $N=2$ SUSY theories may lead to monopole condensation and confinement. We have found that supersymmetric black holes with non-vanishing area of the horizon may stabilize the moduli in theories where the potential is proportional to the square of the graviphoton central charge.

In particular, in known models of spontaneous breaking of $N=2$ to $N=1$ SUSY theories, the parameters of the electric and magnetic Fayet–Iliopoulos terms can be considered proportional to electric and magnetic charges of the dyonic black holes. Upon such identification the potential is found to be proportional to the square of the black hole mass. The fixed values of the moduli near the black hole horizon correspond exactly to the minimum of this potential. The value of the potential at the minimum is proportional to the black hole entropy.

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N=2 supersymmetric theories may be viewed as low-energy effective actions describing the non-perturbative dynamics of more fundamental theories. In particular the vacuum structure and dyon spectrum of these theories has been studied by Seiberg and Witten [1] and the effect of BPS monopoles in N=2 supersymmetric Yang-Mills theories was understood. The relevance of generic supersymmetric black holes to low-energy effective theories was not studied yet², however, one may expect that such relation exists. In particular, one could have guessed that the black holes of N=2 theory with one half of supersymmetry unbroken may be somehow relevant to models with spontaneous breaking of N=2 supersymmetry to N=1. We will find indeed that this is the case and that the effect of supersymmetric dyonic black holes is to stabilize the moduli. The choice of the superpotential in such models will be related to the central charge of the graviphoton, i.e. to the black hole mass as the function of moduli and conserved charges of the black hole.

The main difference between our study of the black holes and the study of non-perturbative phenomena in N=2 theories of rigid supersymmetries in [1] is in the properties of the relevant central charge.

In ref. [1] the central charge defining the mass of the dyon in rigid supersymmetric theory is defined by charge of the vector multiplet. It is defined by the symplectic section of a given N=2 theory (X^Λ, F_Λ) and by conserved charges (q_Λ, p^Λ) of the dyon and is given by

$$(M_{\text{YM}}^{\text{dyon}})^2 = |Z^{\text{rigid}}(t, \bar{t}, q, p)|^2 = \left| X^\Lambda(t) q_\Lambda - F_\Lambda(t) p^\Lambda \right|^2. \quad (1)$$

This formula defines the mass of the BPS dyons in supersymmetric Yang-Mills theory. In our case the central charge of the gravitational multiplet is the charge of the graviphoton, supersymmetric partner of the graviton in theories with local supersymmetry. It is defined as follows [3]:

$$(M_{\text{bh}}^{\text{dyon}})^2 = |Z^{\text{local}}(t, \bar{t}, q, p)|^2 = \left| e^{\frac{K(t, \bar{t})}{2}} [X^\Lambda(t) q_\Lambda - F_\Lambda(t) p^\Lambda] \right|^2, \quad (2)$$

where $K(t, \bar{t})$ is a Kahler potential

$$K = -\ln i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda). \quad (3)$$

This formula for the values of the moduli (t, \bar{t}) at infinity, far away from the black hole, defines the ADM mass of the supersymmetric black hole dyon. In all cases of supersymmetric black holes with non-vanishing area of the horizon and Bertotti-Robinson type geometry near the black hole horizon the following has been proved [4]: The extremum of the square of the graviphoton central charge (2) in the moduli space relates the values of moduli to the ratios of electric and magnetic charges. As a result, the value of the square of the central charge at the extremum in moduli space defines the area of the black hole horizon, which depends only on conserved charges. The extremal value of this mass is also related to the size of the infinite throat of the Bertotti-Robinson geometry and is independent on the values of moduli far away from the horizon:

$$\left(|Z(t, \bar{t}(q, p))|^2 \right)_{\frac{\partial |Z(t, \bar{t}(q, p))|}{\partial t} = 0} = \frac{1}{\pi} S(q, p). \quad (4)$$

²Except for the notable case of black holes which could become massless [2].

We will show in what follows that the idea of the supersymmetric attractors [5], [4] which explains that *moduli are driven to a fixed point of attraction near the black hole horizon*, may be realized in effective field theories in a standard way: *scalars take the values prescribed by the minimum of the potential*. For this to happen we need 3 conditions to be satisfied:

i) to get the potential depending on moduli t, \bar{t} and some parameters (E, M) to be proportional to the square of the central charge, depending on the same moduli t, \bar{t} and black hole charges (q, p) .

$$V(t, \bar{t}, (E, M)) \sim |Z^{\text{local}}(t, \bar{t}, (q, p))|^2 = \left| e^{\frac{K(t, \bar{t})}{2}} [X^\Lambda(t) q_\Lambda - F_\Lambda(t) p^\Lambda] \right|^2, \quad (5)$$

ii) the parameters (E, M) in the potential have to be proportional to the black hole charges

$$(E, M) \sim (q, p), \quad (6)$$

iii) the relevant supersymmetric black hole has to have finite area of the horizon for the potential to have a stable minimum with regular values of moduli.

As we will show, these three conditions are satisfied in a specific model to be discussed below. However one may hope that they could be satisfied in more general situations.

It is important that the graviphoton central charge due to special properties of local supersymmetry, which is doubled near the black hole horizon, has the ability to stabilize the moduli in all cases of supersymmetric configurations with finite area of the horizon. This follows from the universality of the entropy-area formula for supersymmetric black holes proved in [4]. It is not clear to us whether without black holes using only dyonic solutions of rigid supersymmetric theories and the central charge formula (1) one can stabilize the moduli. At the same time various examples of stabilization of moduli near the black hole horizon are known. Still no relation of any black hole to superpotentials in low-energy theories has been established so far.

Spontaneous partial breaking of N=2 SUSY to N=1 was discovered recently in the context of globally supersymmetric theories by Antoniadis, Partouche and Taylor [6]. In what follows we will refer to this theory as APT model. The partial supersymmetry breaking was also found by a suitable flat limit of local N=2 supergravity models by Ferrara, Girardello and Porrati [7], to which we will refer as to FGP model. The APT-FGP potential has a stable minimum, however the mechanism of partial spontaneous supersymmetry breaking does not seem to have a natural dynamical explanation.

The new piece of information about black holes, to be presented below, is the calculation of the attractor values of the axion and dilaton for SL(2,Z) symmetric black holes [8], which was not performed before. This calculation triggered the identification of the APT potential and its minimum with the axion-dilaton black hole near horizon, giving us a first realization of the potential for moduli related to black holes as suggested in eq. (5). The parameters (E, M) in the potential turn out to be the parameters of electric and magnetic Fayet-Iliopoulos terms and they

will be related to black hole charges via the scale of supersymmetry breaking

$$(E, M) = \Lambda^2(q, p) . \quad (7)$$

We will start with the black hole side first. From all known black holes it is the family of axion-dilaton black holes with the manifest $SL(2, Z)$ symmetry [8] which turns out to be relevant to the APT-FGP mechanism. We will first consider the extreme axion-dilaton $U(1) \times U(1)$ black holes of $N=4$ supergravity [8]. The corresponding black holes have been also recognized as supersymmetric black holes of $N=2$ theory interacting with one vector multiplet [5], [8]. There are two versions of this theory, one corresponding to an $SO(4)$ supergravity, which is described by the prepotential $F = -iX^0X^1$. The other one, corresponding to the $SU(4)$ version of $N=4$ theory is related by a symplectic transformation to the first one and has no prepotential but is defined by a symplectic section [3]. In all three versions of these theories one can study supersymmetric black holes and their behavior near the horizon. Alternatively, one can simply find the central charge matrix as a function of moduli and conserved quantized charges and find the extremum of the largest eigenvalue of the central charge [4]. This will define the fixed values of the moduli near the black hole horizon as a function of charges.

N=4 axion-dilaton black holes

We will find the values of the dilaton and axion near the horizon directly from the $U(1) \times U(1)$ $SL(2, Z)$ invariant black hole solutions [8] as functions of conserved electric and magnetic charges in two $U(1)$ groups

$$n_1, m_1 , \quad n_2, m_2 .$$

The eigenvalues of the central charge matrix are given in [8]:

$$Z_1 = \sqrt{2}(\Gamma_1 + i\Gamma_2) , \quad Z_2 = \sqrt{2}(\Gamma_1 - i\Gamma_2) . \quad (8)$$

where

$$\Gamma_1 = \frac{1}{2}(Q_1 + iP_1) , \quad \Gamma_2 = \frac{1}{2}(Q_1 - iP_1) . \quad (9)$$

As found in eq. (30) of [8] in terms of moduli and quantized charges we have

$$\begin{aligned} Q_1 + iP_1 &= e^\phi(n_1 - \bar{\lambda}m_1) , \\ Q_2 + iP_2 &= e^\phi(n_2 - \bar{\lambda}m_2) , \end{aligned} \quad (10)$$

where the axion-dilaton field is

$$\lambda = a + ie^{-\phi} . \quad (11)$$

To find the values of λ near the horizon we may use few methods. The first is based on the fact that we know all solutions and we may simply pick up the one in which moduli do not change and remain constant all the way from the horizon to infinity. Such solution with unconstrained

four charges n_1, m_1, n_2, m_2 exists only in case that the moduli depend on these charges. In our attractor picture presented in Fig. 1 of [4] this corresponds to a solution which is given by the horizontal line with zero slope. The moduli and the metric for the extreme black hole solution is [8]

$$\lambda(x) = \frac{H_1(x)}{H_2(x)} \quad ds^2 = (2\text{Im}H_1H_2)^{-1}dt^2 - 2\text{Im}H_1H_2(d\vec{x})^2, \quad (12)$$

where H_1, H_2 are harmonic functions. The solution under discussion is the one with the vanishing axion-dilaton charge Υ :

$$\bar{\Upsilon} = -\frac{Z_1Z_2}{M} = 0. \quad (13)$$

It follows from eqs. (8), (9), (13) that

$$Q_1^1 + Q_2^2 - P_1^2 - P_2^2 + i(Q_1P_1 + Q_2P_2) = 0. \quad (14)$$

The axion-dilaton field becomes

$$\lambda(r) = \left(\frac{\frac{e^{\phi_0}}{\sqrt{2}} \left[\lambda_0 + \frac{\lambda_0 M + \bar{\lambda}_0 \Upsilon}{r} \right]}{\frac{e^{\phi_0}}{\sqrt{2}} \left[1 + \frac{M + \Upsilon}{r} \right]} \right)_{\Upsilon=0} = \frac{\lambda_0(1 + \frac{M}{r})}{(1 + \frac{M}{r})} = \lambda_0, \quad (15)$$

where λ_0 is the value of the moduli at infinity. For this solution, however the value at infinity has to coincide with the value at the horizon, which we may find by rewriting eqs. (14) in terms of moduli and conserved charges:

$$e^{2\phi}[(n_1 - \lambda m_1)^2 + (n_2 - \lambda m_2)^2] = 0. \quad (16)$$

This complex equation forces the constant complex moduli of this solution to become a function of charges:

$$(e^{-2\phi})_{\text{fix}} = \frac{|n_2 m_1 - n_1 m_2|}{m_1^2 + m_2^2}, \quad (a)_{\text{fix}} = \frac{n_2 m_2 + n_1 m_1}{m_1^2 + m_2^2}. \quad (17)$$

We may also verify that according to [4] the same values of moduli at the fixed point may be obtained by the extremizing of the square of the largest eigenvalue of the central charge.

$$|Z_1|^2 = 2|\Gamma_1 + i\Gamma_2|^2 = \frac{1}{2}|e^{\phi}(n_1 - \bar{\lambda}m_1) + ie^{\phi}(n_2 - \bar{\lambda}m_2)|^2. \quad (18)$$

It can be checked that the equations for the extremum of the central charge

$$\frac{\partial |Z(\lambda, \bar{\lambda}(n_1, m_1, n_2, m_2))|^2}{\partial \lambda} = 0 \quad (19)$$

are solved by the fixed point values of moduli in eqs. (17). We may also note that the value of the central charge at the extremum is proportional to the black hole entropy, which for these black holes was found in [8], [4] to be equal to

$$S = \pi(|Z_1|^2 - |Z_2|^2) = \pi(|Z_1|^2)_{\text{fix}} = 2\pi|n_1 m_2 - n_2 m_1| . \quad (20)$$

We will proceed with identification of the corresponding values of moduli near the black hole horizon in the two versions of N=2 theory. The details will be presented in a separate publication [9].

N=2 black holes in the theory with the prepotential $F = -iX^0 X^1$ (SO(4) version of N=4)

In the version with the prepotential we have

$$q_0, p^0, \quad q_1, p^1,$$

and the moduli take the values near the horizon

$$(e^{-2\phi})_{\text{fix}} = \frac{|q_0 q_1 + p^0 p^1|}{(q_1)^2 + (p^0)^2} \quad (a)_{\text{fix}} = \frac{p^1 q_1 - q_0 p^0}{(q_1)^2 + (p^0)^2} . \quad (21)$$

N=2 black holes in the theory without prepotential (SU(4) version of N=4)

In the version which has no prepotential we have the charges

$$\hat{q}_0, \hat{p}^0, \quad \hat{q}_1, \hat{p}^1,$$

and the moduli are

$$(e^{-2\hat{\phi}})_{\text{fix}} = \frac{|\hat{q}_1 \hat{p}^0 - \hat{q}_0 \hat{p}^1|}{(\hat{p}^1)^2 + (\hat{p}^0)^2}, \quad (a)_{\text{fix}} = \frac{-\hat{q}_1 \hat{p}^1 - \hat{q}_0 \hat{p}^0}{(\hat{p}^1)^2 + (\hat{p}^0)^2} . \quad (22)$$

For all these cases it is easy to identify the relation between charges of these three versions of the same theory as well as central charges.

Superpotential in APT-FGP model and black holes near the horizon

APT model as well as flat limit of FGP model³ in terms of manifest N=1 supersymmetry consist of usual terms

$$-\frac{i}{4} \int d^2\theta \lambda \mathcal{W} + c.c. + \int d^2\theta d^2\bar{\theta} K, \quad (23)$$

³We use the notation for the F-I parameters as given in FGP model and we consider b^0 , the constant value of one of the quaternionic coordinates of the hypermultiplet manifold, to be equal to 1.

where \mathcal{W} is the gauge field superfield and K is the Kahler potential. This action is supplemented by the F-I term

$$\Lambda^2 \sqrt{2} \xi D , \quad (24)$$

as well as by an unusual superpotential term

$$\Lambda^2 \int d^2 \theta W + c.c. . \quad (25)$$

Here

$$W = eb + m \mathcal{F}_b \quad (26)$$

and b is a chiral superfield and \mathcal{F}_b is the derivative of the prepotential over b . In terms of the manifest N=2 superfields the APT Lagrangian is

$$\frac{i}{4} \int d^2 \theta_1 d^2 \theta_2 [\mathcal{F}(B) - B^D B] + \frac{1}{2} (\vec{E} \cdot \vec{Y} + \vec{M} \cdot \vec{Y}^D) + c.c. , \quad (27)$$

where B is an unconstrained chiral N=2 multiplet, B^D is a constrained chiral N=2 multiplet playing the role of the Lagrange multiplier. The constant vectors \vec{E}, \vec{M} (\vec{M} being real) define electric and magnetic Fayet–Iliopoulos terms, since they are the coefficients in front of the auxiliary fields of the B, B^D multiplets, \vec{Y}, \vec{Y}^D . The N=2 auxiliary fields form SU(2) triplets since they combine the complex auxiliary field of N=1 chiral multiplet $F + iG$ with the auxiliary field of N=1 vector multiplet D . In APT-FGP model the F-I parameters which lead to spontaneous breaking of N=2 supersymmetry to N=1 are chosen to be

$$\text{Re} \vec{E} = \Lambda^2(0, e, \xi) , \quad \vec{M} = \Lambda^2(0, m, 0) . \quad (28)$$

Upon elimination of auxiliary fields the scalar potential is

$$V = \frac{|\text{Re} \vec{E} + \lambda \vec{M}|^2}{\text{Im} \lambda} + \dots , \quad (29)$$

where \dots denote terms independent of moduli λ . These terms differ in APT model and in the flat limit of FGP model. At the moment from the black hole side we do not have information on field independent part of the potential and in what follows we will compare the APT-FGP field dependent part of the potential with the black hole central charge. A stable minimum of the potential for the scalar fields in APT theory exists at

$$(\text{Im} \lambda)_{\min} = (e^{-2\phi})_{\min} = \left| \frac{\xi}{m} \right| , \quad (\text{Re} \lambda)_{\min} = (a)_{\min} = -\frac{e}{m} . \quad (30)$$

It is fairly easy to see that if one would take any of the 3 types of black hole solutions above and in each case choose only 3 non-vanishing charges one would always reproduce the relevant values of the scalars at the minimum of the potential from the fixed points of moduli in black holes. In particular, in the first case we may take a black hole with

$$m_2 = 0 , \quad (e^{-2\phi})_{\text{fix}} = \left| \frac{n_2}{m_1} \right| = \left| \frac{\xi}{m} \right| = (e^{-2\phi})_{\min} , \quad (a)_{\text{fix}} = \frac{n_1}{m_1} = -\frac{e}{m} = (a)_{\min} . \quad (31)$$

In the second case

$$p^0 = 0, \quad (e^{-2\phi})_{\text{fix}} = \left| \frac{q_0}{q_1} \right| = \left| \frac{\xi}{m} \right| = (e^{-2\phi})_{\text{min}}, \quad (a)_{\text{fix}} = \frac{p^1}{q_1} = -\frac{e}{m} = (a)_{\text{min}}. \quad (32)$$

In the third case we need

$$\hat{p}^1 = 0, \quad (e^{-2\phi})_{\text{fix}} = \left| \frac{\hat{q}_1}{\hat{p}^1} \right| = \left| \frac{\xi}{m} \right| = (e^{-2\phi})_{\text{min}}, \quad (a)_{\text{fix}} = \frac{\hat{q}_0}{\hat{p}^1} = -\frac{e}{m} = (a)_{\text{min}}. \quad (33)$$

This gives the relation between the ratios of parameters of F-I terms leading to spontaneous breaking of N=2 supersymmetry to N=1 supersymmetry in APT-FGP model and ratios of charges of supersymmetric black holes with 1/4 of unbroken N=4 supersymmetry or 1/2 of N=2 one.

Let us now compare the APT-FGP potential (29) with the square of the graviphoton central charge. The field dependent part of the potential is

$$V \sim e^{2\phi} [(e + ma)^2 + (e^{-\phi}m + \xi)^2] + \dots, \quad (34)$$

and its value at infinity is

$$V_{\text{min}} \sim 2|m\xi|. \quad (35)$$

We take the N=4 black hole central charge (18) at $m_2 = 0$, i.e. restricting the 4-charge solution to only 3-charge solution. We get

$$|Z|^2 = \frac{1}{2}e^{2\phi} [(n_1 - am_1)^2 + (m_1e^{-\phi} + n_2)^2], \quad (36)$$

and its value at the fixed point is

$$|Z|_{\text{fix}}^2 = 2|m_1n_2|. \quad (37)$$

Relation (31) between F-I parameters of the APT-FGP model and black hole charges,

$$\left| \frac{n_2}{m_1} \right| = \left| \frac{\xi}{m} \right|, \quad \frac{n_1}{m_1} = -\frac{e}{m},$$

which was found before from the minimal values of the potential and from the fixed point of the values of the moduli near black hole horizon is confirmed. The same can be established for other cases. With such identification the potential coincides with the black hole central charge as a function of moduli in the generic point of the moduli space. It is quite remarkable that the procedure of variation of the central charge over the moduli at fixed conserved charges of the black hole suggested in [4] becomes the procedure of variation of the potential over the scalar fields at the fixed values of the F-I parameters. The minimum of the field dependent part of the potential is proportional to the minimum of the central charge, i.e. proportional to the black hole entropy.

$$V_{\text{min}}(E, M) \sim \pi(|Z(q, p)|^2)_{\text{fix}} = S(q, p). \quad (38)$$

In conclusion, we have found an interesting correspondence between specific Lorentz-covariant models of $N=2$ supersymmetry spontaneously broken down to $N=1$ [6, 7] and the axion-dilaton black holes: the field dependent part of the potential is proportional to the square of the black hole central charge, considered in the generic point of moduli space. The minimum of the potential is the minimum of the central charge extremized in the moduli space. According to [4] this minimum describes the stabilization of the moduli near the black hole horizon. The APT-FGP model clearly shows that the origin of the superpotential stabilizing the moduli is related to supersymmetric black holes with non-vanishing area of the horizon. It is also important to stress that whereas black holes require the concept of static geometry breaking Lorentz invariance, the low-energy action with partially spontaneously broken supersymmetry describes Lorentz-covariant theory which codifies the most important properties of extreme black holes with partially broken supersymmetry. In particular, the value of the potential at the minimum is proportional to the black hole entropy. In [1] the effects of non-gravitational dyons (solitons of gauge theories) on supersymmetric low-energy theories were studied. In our case of gravitational dyons (black holes) a new phenomenon which was not seen before is the stabilization of the moduli as a generic property of black holes with non-vanishing entropy.

Our observation opens various directions for future investigation. During the last few years many new black hole solutions which break $1/2$, $3/4$ or $7/8$ of the original supersymmetry have been found. It would be very interesting to find the counterparts of these solutions in supersymmetric models of elementary particles. It seems likely that new models of global supersymmetry may be constructed which are related to generic supersymmetric black holes in such a way that the potential is proportional to the square of the black hole central charge. Such models will have build-in duality symmetry, since the black hole mass formulae have such symmetry. It may be possible, in particular, to construct models with global $N=4$ supersymmetry with the black hole type potential for breaking the supersymmetry spontaneously down to $N=1$, since there exist black holes which break $3/4$ of $N = 4$ supersymmetry. One may try to use the fact that theories with global $N=4$ supersymmetry and some $N=2$ theories are finite and therefore the new models of $N=1$ supersymmetry have to be finite, since the breaking of supersymmetry from $N=4$ and from $N=2$ to $N=1$ in such theories will be spontaneous.

It would be interesting to combine the picture discussed above with hypermultiplets to include massless black holes and to find the relation of such models to string theory and their vacua. It would be especially important to understand the dynamical mechanism behind the relation between the potential leading to partial spontaneous breaking of supersymmetry and the black hole mass formula.

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